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Navigation, Control, and Guidance Theory; Computer Technology and Computer Simulation Techniques

Theme

A new statistical evaluation technique has been developed at the University of Wisconsin which offers a method of resolving uncertainties in mathematical model formulation. The advantages of the technique are: 1) no model assumptions required prior to processing the data; 2) deterministic processes are positively identified by the analysis technique; 3) the technique estimates the parameters of the processes; and 4) the technique measures the goodness of the model. The theory has recently been advanced to include the capability of determining the presence of two different nonstationary processes in the data.

Content

All of the methods presently being used for time series math modelling assume the presence of some specific process and proceed to estimate the parameters of the process. The most basic problem is the fact that there are many models with various combinations of deterministic and stochastic parameters which can fit the data. Although the data can be adequately represented by any one or more of these models, the model may not truly represent the process which produced it.

In this technique, the data is treated as a purely random process, and the systematic application of time series techniques in conjunction with pattern recognition determines the time series model. If deterministic phenomena exist in the data, these will be directly evident from the analysis without making a priori assumptions regarding their existence. The development of a math model consists of a three-stage iterative process consisting of identification, estimation, and diagnostic checking. Identification means using the data in the light of the information on how the series was generated to suggest a class of models that should be considered. Estimation means using the data to make inferences about parameters conditional on the adequacy of the model chosen. Diagnostic checking means checking the fitted model in relation to the data with the hope of revealing model inadequacies and thus proceeding to model improvement.

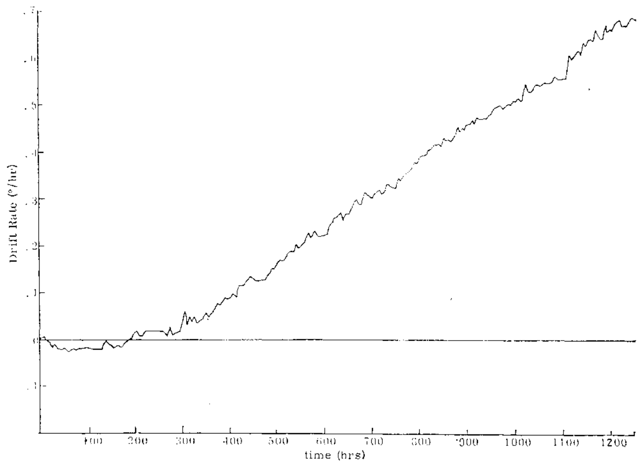

Fig. 1 Normalized gyro drift rate vs time: 1250-hr drift test.

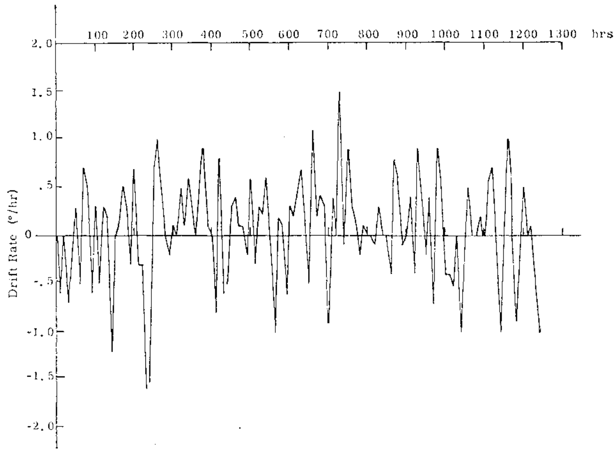

Fig. 2 Differenced gyro drift rate vs time.

The equation for a linear stationary process is given by

$$x_t = \phi_1 x_{t-1} + \ldots + \phi_p x_{t-p} + z_t - \theta_1 z_{t-1} - \ldots \theta_q Z_{t-q} \quad (1)$$

where x_t = the time series data (output random process); z_t = white noise (input random process); ϕ = autoregressive parameters; θ = moving average parameters.

Thus the general form for the mixed autoregressive moving average linear stationary process can be expresses as

$$\phi(B)x_t = \theta(B)z_t \quad (2)$$

where the ϕ and θ parameters were previously defined and B is a backward shift operator such that $Bx_t = x_{t-1}$ and $Bz_t = z_{t-1}$.

If the data were comprised of a nonstationary process (ramp and/or random walk) in addition to a stationary process, it would be necessary to difference the data to induce

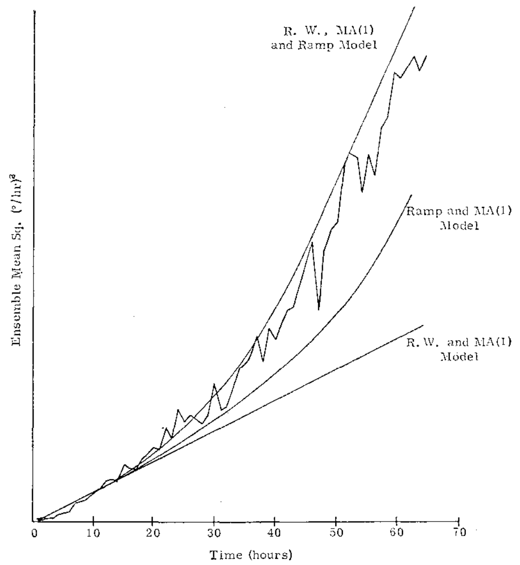

Fig. 3 Normalized ensemble mean squared gyro drift rate vs time.

stationarity, since the autoregressive and moving average processes are both linear models being fit to stationary processes. Data differencing would be of the form

$$x_t - x_{t-1} = (1 - B)x_t \tag{3}$$

The full text of the paper describes in detail: 1) the identification of the time series model from autocorrelation and partial correlation of the data; 2) the estimation of the ϕ, θ, ramp and random walk parameters using maximum likelihood and nonlinear least squares; and 3) the means by which one would deduce model adequacy through autocorrelation of the white noise residuals and confidence limit theory.

As an example of the theory, consider Fig. 1, which shows a sample of normalized long term gyro drift rate. Since the process is nonstationary, the data is differenced as is shown in Fig. 2. The analysis indicated that the math model for this gyro drift rate sample is

$$x_t = x_{t-1} - \theta_1 z_{t-1} + z_t + b(1 - \theta_1) \tag{4}$$

where θ_1 = first-order moving average parameter, $b(1 - \theta_1)$ = mean of the differenced data indicating a ramp in the original process, and z_t = white noise residual indicating a random walk process in the original data. The mean squared value of Eq. (4) is given as

$$\overline{x_t^2} = n\sigma_z^2(1 - \theta_1)^2 + n^2b^2(1 - \theta_1)^2 + 2\theta^2\sigma_z^2 \tag{5}$$

where $E[z_t^2] = \sigma_z^2$. Note that this equation consists of random walk, ramp, and stationary noise processes. A picture of model adequacy is shown in Fig. 3. The data of Fig. 1 was broken into 64-hr segments and ensemble averaged. Using the appropriate values of θ_1, b, and σ_z^2 determined from the analysis, various combinations of Eq. (5) were superimposed on Fig. 3. It can be seen that model adequacy is attained only by using the combination of two nonstationary processes and the one stationary process. A good fit of the data is not attained with only one of the nonstationary processes.

It is important to note that the models and parameter values obtained were not from the ensemble mean squared gyro drift rate waveform, but from the single sample long term gyro drift rate process.

Stationary and Nonstationary Characteristics of Gyro Drift Rate

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Procedures for the mathematical modeling of gyro drift rate based on modeling techniques for stationary and nonstationary time series analysis are presented. The essence of these new time series analysis methods is the reduction of a random process to white noise (uncorrelated) residuals. The math model of the time series is established by reducing the wave form to white noise while identifying the correlated portion of the time series. This approach differs from other known techniques in that no deterministic models of the time series are initially assumed, e.g., ramps or sinusoids. The data is treated as a random process on which systematic application of the time series techniques will determine the exact gyro drift rate math model. If deterministic phenomena exist in the data, these will be directly evident from the analysis without making a priori assumptions as to their existence. Once the time series math model has been identified, parameter estimation techniques for the numerical evaluation of the math model are employed. Application of the single sample gyro drift rate model to an ensemble average leads to model verification.

1. Introduction

OVER the past several years the investigation of a mathematical model for gyro drift rate has assumed increased importance. The drift rate of a gyro is a major error source of inertial navigation systems that are required to operate over long time intervals, and as such, must be mathematically modeled whenever an error analysis or optimization study is performed on the navigation system.

Of particular importance as far as long term navigation is concerned, is the nonstationary behavior of gyro drift rate. In order to minimize the buildup of navigation errors by use of a Kalman filter, an accurate math model of the nonstationary gyro drift rate in the filter is essential in order to provide as near optimal control to the system as possible.

The gyros used in long term inertial navigation systems must remain in operation over a period of many months. In order to gain insight into the behavior of gyros in an inertial system over prolonged time periods, gyros were tested for time periods of 500–1000 hr. It is to this aspect of gyro drift rate modeling that this paper is addressed.

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